

Iterationsverfahren

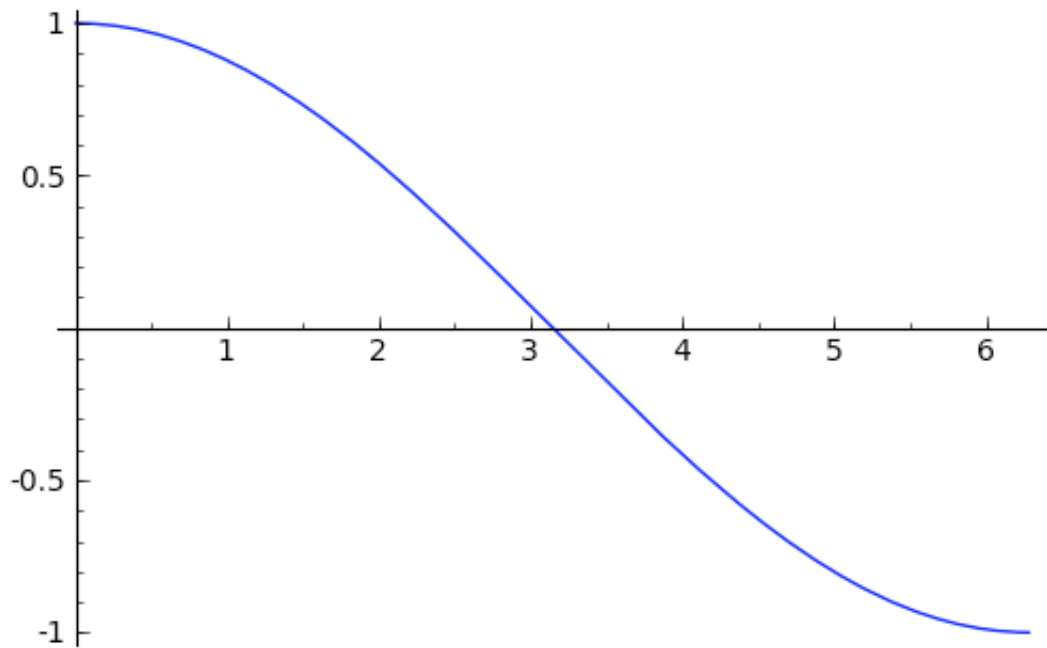
Aufgabe 6.1(a) (b) bitte selbst probieren!)

mit Bisektionsverfahren

```
f(x)=cos(x/2); f(x)
```

```
cos(1/2*x)
```

```
plot(f(x),(x,0,2*pi))
```



```
eps=0.000001
a=2.5
b=3.5
t=(a+b)/2.
print "a =",a
print "b =",b
print "f(a) =",f(a)
print "f(b) =",f(b)
print "Näherung t =",t
print "b-t =", b-t
print "f(t) =",f(t)
print " "
while b-t>eps:
    if f(a)*f(t)<0:
        b=t
    else:
        a=t
```

```

t=(a+b)/2
print "a =",a
print "b =",b
print "f(a) =",f(a)
print "f(b) =",f(b)
print "Näherung t =",t
print "b-t =", b-t
print "f(t) =",f(t)
print " "

```

WARNING: Output truncated!
[full_output.txt](#)

```

a = 2.5000000000000000
b = 3.5000000000000000
f(a) = 0.315322362395269
f(b) = -0.178246055649492
Näherung t = 3.0000000000000000
b-t = 0.5000000000000000
f(t) = 0.0707372016677029

```

```

a = 3.0000000000000000
b = 3.5000000000000000
f(a) = 0.0707372016677029
f(b) = -0.178246055649492
Näherung t = 3.2500000000000000
b-t = 0.2500000000000000
f(t) = -0.0541771350269363

```

```

a = 3.0000000000000000
b = 3.2500000000000000
f(a) = 0.0707372016677029
f(b) = -0.0541771350269363
Näherung t = 3.1250000000000000
b-t = 0.1250000000000000
f(t) = 0.00829623162385838

```

```

a = 3.1250000000000000
b = 3.2500000000000000
f(a) = 0.00829623162385838
f(b) = -0.0541771350269363
Näherung t = 3.1875000000000000
b-t = 0.0625000000000000
f(t) = -0.0229516576536404

```

```

a = 3.1250000000000000
b = 3.1875000000000000
f(a) = 0.00829623162385838
f(b) = -0.0229516576536404

```

```
Näherung t = 3.156250000000000
b-t = 0.03125000000000000
f(t) = -0.00732860760211055
```

```
a = 3.1250000000000000
b = 3.1562500000000000
f(a) = 0.00829623162385838
f(b) = -0.00732860760211055
Näherung t = 3.140625000000000
b-t = 0.01562500000000000
f(t) = 0.000483826776020249
```

```
a = 3.1406250000000000
b = 3.1562500000000000
f(a) = 0.000483826776020249
f(b) = -0.00732860760211055
Näherung t = 3.148437500000000
b-t = 0.00781250000000000
f(t) = -0.00342241652397786
```

```
a = 3.1406250000000000
b = 3.1484375000000000
f(a) = 0.000483826776020249
```

...

```
Näherung t = 3.14147949218750
b-t = 0.000122070312500000
f(t) = 0.0000565807011164299
```

```
a = 3.14147949218750
b = 3.14160156250000
f(a) = 0.0000565807011164299
f(b) = -4.45445510336604e-6
Näherung t = 3.14154052734375
b-t = 0.000061035156250000
f(t) = 0.0000260631230186685
```

```
a = 3.14154052734375
b = 3.14160156250000
f(a) = 0.0000260631230186685
f(b) = -4.45445510336604e-6
Näherung t = 3.14157104492188
b-t = 0.0000305175781250000
f(t) = 0.0000108043339589090
```

```
a = 3.14157104492188
b = 3.14160156250000
f(a) = 0.0000108043339589090
f(b) = -4.45445510336604e-6
Näherung t = 3.14158630371094
```

```

b-t = 0.0000152587890625000
f(t) = 3.17493942786390e-6

a = 3.14158630371094
b = 3.14160156250000
f(a) = 3.17493942786390e-6
f(b) = -4.45445510336604e-6
Näherung t = 3.14159393310547
b-t = 7.62939453125000e-6
f(t) = -6.39757837755725e-7

a = 3.14158630371094
b = 3.14159393310547
f(a) = 3.17493942786390e-6
f(b) = -6.39757837755725e-7
Näherung t = 3.14159011840820
b-t = 3.81469726562500e-6
f(t) = 1.26759079505639e-6

a = 3.14159011840820
b = 3.14159393310547
f(a) = 1.26759079505639e-6
f(b) = -6.39757837755725e-7
Näherung t = 3.14159202575684
b-t = 1.90734863281250e-6
f(t) = 3.13916478650476e-7

a = 3.14159202575684
b = 3.14159393310547
f(a) = 3.13916478650476e-7
f(b) = -6.39757837755725e-7
Näherung t = 3.14159297943115
b-t = 9.53674316406250e-7
f(t) = -1.62920679552643e-7

```

[full_output.txt](#)

N(pi)

3.14159265358979

mit Newton-Verfahren

```

f(x) = cos(x/2); print "f(x)=",f(x)
New(x) = x - f(x)/diff(f(x),x); print "New(x)=", New(x)

f(x)= cos(1/2*x)
New(x)= x + 2*cos(1/2*x)/sin(1/2*x)

```

```

eps=0.000001
xi = 3.0
print "x0=",xi

```

```

print "f(x0)=",f(xi)
print ""
xii=New(xi)
print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)
print "f(xn)=",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = New(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)=",f(xii)
    print ""

```

```

x0= 3.000000000000000
f(x0)= 0.0707372016677029

```

```

xn= 3.14182968860530
abs(xn-x(n-1))= 0.141829688605305
f(xn)= -0.000118517507478303

```

```

xn= 3.14159265358868
abs(xn-x(n-1))= 0.000237035016621423
f(xn)= 5.54950700047611e-13

```

```

xn= 3.14159265358979
abs(xn-x(n-1))= 1.10977893541531e-12
f(xn)= 6.12323399573677e-17

```

N(pi)

```
3.14159265358979
```

mit Regula falsi

```

f(x) =cos(x/2); print "f(x)=",f(x)
h=0.01
Nu(x) = x - f(x)/((1/h)*(f(x+h)-f(x))); print "Nu(x)=", Nu(x)
f(x)= cos(1/2*x)
Nu(x)= x - cos(1/2*x)/(100.000000000000*cos(1/2*x +
0.005000000000000000) - 100.000000000000*cos(1/2*x))

```

```

eps=0.000001
xi = 1.0
print "x0=",xi
print "f(x0)=",f(xi)
print ""
xii=Nu(xi)
print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)

```

```

print "f(xn)=",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = Nu(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)=",f(xii)
    print ""

```

```

x0= 1.000000000000000
f(x0)= 0.877582561890373

```

```

xn= 4.64431348506843
abs(xn-x(n-1))= 3.64431348506843
f(xn)= -0.682633530004910

```

```

xn= 2.77165082812692
abs(xn-x(n-1))= 1.87266265694152
f(xn)= 0.183917942519529

```

```

xn= 3.14569688804780
abs(xn-x(n-1))= 0.374046059920885
f(xn)= -0.00205211578869621

```

```

xn= 3.14159260967128
abs(xn-x(n-1))= 0.00410427837652216
f(xn)= 2.19592575846952e-8

```

```

xn= 3.14159265358998
abs(xn-x(n-1))= 4.39186980116801e-8
f(xn)= -9.14211448891555e-14

```

N(pi)

```
3.14159265358979
```

Aufgabe 6.2

```

f(x) = (x-1)^2/(x^2+1); print "f(x)=",f(x)
New(x) = x - f(x)/diff(f(x),x); print "New(x)=", New(x)

```

```

f(x)= (x - 1)^2/(x^2 + 1)
New(x)= x + 1/2*(x - 1)^2/((x^2 + 1)*((x - 1)^2*x/(x^2 + 1)^2 -
1)/(x^2 + 1))

```

```

eps=0.0001
xi = 0.0
print "x0=",xi
print "f(x0)=",f(xi)
print ""
xii=New(xi)

```

```

print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)
print "f(xn)=",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = New(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)=",f(xii)
    print ""

```

```

x0= 0.0000000000000000
f(x0)= 1.0000000000000000

```

```

xn= 0.5000000000000000
abs(xn-x(n-1))= 0.5000000000000000
f(xn)= 0.2000000000000000

```

```

xn= 0.7083333333333333
abs(xn-x(n-1))= 0.2083333333333333
f(xn)= 0.0566473988439307

```

```

xn= 0.836530318428184
abs(xn-x(n-1))= 0.128196985094851
f(xn)= 0.0157210286297997

```

```

xn= 0.912179214244263
abs(xn-x(n-1))= 0.0756488958160788
f(xn)= 0.00420971171541152

```

```

xn= 0.954250037538292
abs(xn-x(n-1))= 0.0420708232940287
f(xn)= 0.00109550224369870

```

```

xn= 0.976614003890492
abs(xn-x(n-1))= 0.0223639663522006
f(xn)= 0.000279922119230973

```

```

xn= 0.988171893396200
abs(xn-x(n-1))= 0.0115578895057075
f(xn)= 0.0000707843461978079

```

```

xn= 0.994051178752327
abs(xn-x(n-1))= 0.00587928535612736
f(xn)= 0.0000177998100525673

```

```

xn= 0.997016768651073
abs(xn-x(n-1))= 0.00296558989874562
f(xn)= 4.46312932774835e-6

```

```
xn= 0.998506162731895
abs(xn-x(n-1))= 0.00148939408082249
f(xn)= 1.11744292285765e-6
```

```
xn= 0.999252523895510
abs(xn-x(n-1))= 0.000746361163614484
f(xn)= 2.79569156555501e-7
```

```
xn= 0.999626122319846
abs(xn-x(n-1))= 0.000373598424336707
f(xn)= 6.99183958993900e-8
```

```
xn= 0.999813026220327
abs(xn-x(n-1))= 0.000186903900480817
f(xn)= 1.74828656744279e-8
```

```
xn= 0.999906504371182
abs(xn-x(n-1))= 0.0000934781508549198
f(xn)= 4.37112496598793e-9
```

$x_0=0$: lineare Konvergenz (die gesuchte Nullstelle ist eine doppelte Nullstelle).

```
eps=0.0001
xi = -2.
print "x0=",xi
print "f(x0)",f(xi)
print ""
xii=New(xi)
print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)
print "f(xn)",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = New(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)",f(xii)
    print ""
```

```
x0= -2.000000000000000
f(x0)= 1.800000000000000
```

```
xn= -9.500000000000001
abs(xn-x(n-1))= 7.500000000000001
f(xn)= 1.20821917808219
```

```
xn= -65.8602941176472
abs(xn-x(n-1))= 56.3602941176472
```

```
f(xn)= 1.03036031111448
```

```
xn= -2302.04059437296
abs(xn-x(n-1))= 2236.18030025532
f(xn)= 1.00086879424606
```

```
xn= -2.65430103112789e6
abs(xn-x(n-1))= 2.65199899053352e6
f(xn)= 1.00000075349404
```

```
xn= -3.52266229210863e12
abs(xn-x(n-1))= 3.52265963780760e12
f(xn)= 1.000000000000057
```

```
xn= -6.20132768584489e24
abs(xn-x(n-1))= 6.20132768584137e24
f(xn)= 1.000000000000000
```

```
xn= 2.17780714829401e40
abs(xn-x(n-1))= 2.17780714829401e40
f(xn)= 1.000000000000000
```

Traceback (click to the left of this block for traceback)

...

RuntimeError: power::eval(): division by zero

x_0=-2: die Näherungen x_n entfernen sich von der gesuchte Nullstelle.

```
eps=0.0001
xi = -1.
print "x0=",xi
print "f(x0)",f(xi)
print ""
xii=New(xi)
print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)
print "f(xn)",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = New(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)",f(xii)
    print ""
```

```
x0= -1.000000000000000
f(x0)= 2.000000000000000
```

Traceback (click to the left of this block for traceback)

...

RuntimeError: power::eval(): division by zero

$x_0 = -1$: $f'(-1) = 0$.

Aufgabe 6.3.

```
f(x) = x^2-42; print "f(x)=",f(x)
New(x) = x - f(x)/diff(f(x),x); print "New(x)=", New(x)
```

```
f(x)= x^2 - 42
New(x)= x - 1/2*(x^2 - 42)/x
```

```
eps=0.000001
xi = 6.0
print "x0=",xi
print "f(x0)=",f(xi)
print ""
xii=New(xi)
print "xn=",xii
print "abs(xn-x(n-1))=",abs(xii-xi)
print "f(xn)=",f(xii)
print ""
while abs(xii-xi)>eps:
    xi=xii
    xii = New(xi)
    print "xn=",xii
    print "abs(xn-x(n-1))=",abs(xii-xi)
    print "f(xn)=",f(xii)
    print ""
```

```
x0= 6.000000000000000
f(x0)= -6.000000000000000
```

```
xn= 6.500000000000000
abs(xn-x(n-1))= 0.500000000000000
f(xn)= 0.250000000000000
```

```
xn= 6.48076923076923
abs(xn-x(n-1))= 0.0192307692307692
f(xn)= 0.000369822485204452
```

```
xn= 6.48074069847067
abs(xn-x(n-1))= 0.0000285322985620340
f(xn)= 8.14090128642420e-10
```

```
xn= 6.48074069840786
abs(xn-x(n-1))= 6.28084251275141e-11
f(xn)= 0.000000000000000
```

`N(sqrt(42))`

`6.48074069840786`